A VALUATION MODEL FOR INDETERMINATE CONVERTIBLES

by
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Abstract

Many issues of convertible debentures in India in recent years provide for a mandatory conversion of the debentures into an unspecified number of shares at an unspecified time; the conversion ratio (i.e., the number of shares per debenture) is to be determined by the Controller of Capital Issues (CCI). There are serious problems in arriving at a rational value for these "indeterminate convertibles". Even if the investor can make some estimate of the likely conversion terms, there is no valuation model available to arrive at a price. This paper applies the general theory of derivative securities (Cox, Ingersoll and Ross, 1985) to obtain a valuation model for these instruments. The model shows that the naive valuation model which sets the value of the debenture equal to the current stock price times the expected conversion ratio is likely to be a significant overestimate of the price. It also shows that changes in the stock price lead to less than proportionate changes in the debenture price unlike in the case of pre-specified conversion terms. Similarly, the CAPM beta of the debenture would be significantly lower than that of the share. While the model does not obviate the need for obtaining estimates of unobservable parameters related to the market expectations about the likely conversion ratio, the qualitative insights given by the model are quite useful. The model is successful in explaining some of the empirical patterns and anomalies that have been observed in ongoing empirical research into the market prices of these debentures.

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Introduction

Convertible debentures are debt instruments to begin with, but are subsequently converted, wholly or partly, into equity. Such instruments are well known throughout the world. In most countries, however, the conversion is at the option of the investor and the terms of conversion are clearly specified at the time of the issue. In India too, many convertible debentures issued in the early eighties were of this kind. In recent years, the conversion has become mandatory: the investor has lost the option to forego conversion and hold the debt instrument till redemption. What is worse is that the conversion terms are no longer specified at the time of issue, but are left to be determined by the Controller of Capital Issues (CCI). With this has arisen a security which is perhaps unique in the world, a security which is mandatorily converted into equity shares at a unspecified price at an unspecified time according to the essentially arbitrary decision of a government official. In this paper, these securities are referred to as indeterminate convertibles.

While well developed theories exist for valuing the orthodox variety of convertible securities, there is no theory available for valuing this new breed of convertible debentures. This paper applies the general theory of derivative securities to obtain a valuation model for these indeterminate convertibles. The valuation model does contain some unobservable parameters related to the market's expectation of the conversion terms. Nevertheless, it provides valuable insights into the qualitative behaviour of the prices of these securities. In fact, the motivation for this paper arose from ongoing empirical research on the prices of one of the largest issues of indeterminate convertibles in the Indian capital markets. In the course of that empirical research, a number of empirical patterns and anomalies were observed, and a need was felt for theoretical models that could explain these phenomena at least at a qualitative level. In our opinion, our model achieves this purpose.

The Cox Ingersoll Ross Model

The basic model for valuing derivative securities is that of Cox, Ingersoll and Ross (1985) henceforth referred to as CIR. We present below the salient features of the CIR model with some simplifications and in a somewhat altered notation. There are n state variables Y_i which evolve according to a diffusion process:

$$\frac{dY_i}{Y_i} = \mu_i \, dt + \sum_j \sigma_{ij} \, dw_j \qquad (1)$$

where the w_j are a set of standardized Wiener processes. Some of the Y_i may be traded (non derivative) securities, but some may be state variables which are not traded but affect the prices of other (derivative) securities. A derivative security or contingent claim F pays an amount $\Theta(Y(T))$ at time T. In other words, the payoff at time T depends on the values of the state variables Y_i at that time. CIR show that associated with the Y_j are a set of factor risk premiums and the return on F is equal to the riskfree interest rate r plus the risk premiums for each of the factors Y_j:

$$\delta = r + \sum_j \lambda_j \frac{\partial F/F}{\partial Y_j/Y_j} \tag{2}$$

Here δ is the expected return on the security F, λ_j is the risk premium per unit of the j'th factor risk (if a security's payoff is equal to Y_j, its expected return equals $r + \lambda_j$), and the coefficient $(\partial F/F)/(\partial Y_j/Y_j)$ is the measure of the security's exposure to the j'th factor risk (the extent to which returns on Y_j influence returns on F).

The principle of risk neutral valuation asserts that the value of the security F can be computed by the following procedure:

1. Change the system dynamics by reducing the drift of the state variables by the risk adjustment λ_j:

   $$\frac{dY_i}{Y_i} = (\mu_i - \lambda_i)dt + \sum_j \sigma_{ij}\, dw_j \tag{3}$$

   If Y_j is a traded security, Eqn 0 applies to it, $\mu_i = \lambda_i + r$, and the risk adjustment amounts to choosing the dynamics that the security would have in a risk neutral world.

2. Compute the expected payoff of the derivative security F under the above altered dynamics and discount this at the riskfree rate r to get the value of F:

   $$F(t) = \exp(-r(T-t))\, E'_t(\Theta(Y(T)) \tag{4}$$

where E_t denotes the expectation at time t with respect to the altered dynamics.

The above procedure may be summarized as follows :

> "The equilibrium price of a claim is given by its expected discounted value with discounting done at the riskfree rate, where the expectation is taken with respect to a risk adjusted process for wealth and the state variables. The risk adjustment is accomplished by reducing the drift of each underlying variable by the corresponding factor risk premium." (CIR, page 380).

<u>A Simple Valuation Model</u>

The value of the convertible debenture is the sum of two components: the present value of all interest payments receivable till conversion, and the present value of the shares to be received on conversion. The valuation of the interest stream is, in principle, a straightforward computation of present value; the choice of the discount rate is also not too problematic if some form of credit rating is available. This paper, therefore, ignores this component of the value completely by implicitly assuming that the debenture pays no interest at all. In a real life application, the present value of the interest stream would have to be added to the value computed according to the model. For simplicity, we assume that the shares of the company do not pay any dividend either. In most cases, dividend yields are low and a simple adjustment for the present value of estimated dividends should be adequate to use our model in practice.

The component of the value of the convertible arising from the conversion into shares is valued using the CIR model discussed above. In this section, we present a simple model in which we take the price of the share as a state variable. In the next section, we present a more comprehensive model in which the price of the share is itself a derivative security and the total value of the firm is taken as a state variable. The latter model is the more correct one as the value of the share reflects the dilution effect of future conversion of debentures into shares. The value of the share, therefore, depends on the unknown conversion ratio, and should not be taken as a state variable. However, the simpler model of this section is intuitively clearer, and may be adequate if the debenture issue is not so large as to make the dilution effect substantial.

In the model of this section, there are two state variables:

$S(t)$ is the price of the share at time t

$K(t)$ is the market's expectation at time t of the conversion ratio. It is assumed that as t approaches the conversion date T, this expectation converges to the actual conversion ratio so that $K(T)$ is the actual conversion ratio.

There is only one derivative security - the convertible debenture:

$D(t)$ is the price at time t of the debenture

The payoff to the debenture ($\Theta(Y(T))$ in the earlier notation) is simply $S(T)*K(T)$. We let

$$D'(t) = S(t)*K(t) \qquad (5)$$

$D'(T)$ is the payoff to the debenture.

The system dynamics is as follows:

$$\frac{dS}{S} = \mu_S \, dt + \sigma_S \, dw_S \tag{6}$$

$$\frac{dK}{K} = \sigma_K \, dw_K \tag{7}$$

Using Ito's lemma, we can write the dynamics of D' as follows:

$$\frac{dD'}{D'} = (\mu_S + \sigma_{SK})dt + \sigma_S \, dw_S + \sigma_K \, dw_K \tag{8}$$

where $\sigma_{SK} = \sigma_S\sigma_K\rho_{SK}$, ρ_{SK} is the instantaneous correlation between the Wiener processes w_S and w_K so that σ_{SK} can be interpreted as a covariance term. We assume σ_{SK} and ρ_{SK} to be negative. This is because the CCI while fixing conversion ratios tends to look at the wealth transferred to debenture holders; the conversion ratio would be adjusted so that the value of shares that they receive is neither much lower than the face value not much higher than it. To achieve this, the conversion ratio would have to be reduced when the stock price rises and vice versa resulting in a negative correlation.

CIR's risk adjusted system dynamics are as follows:

$$\frac{dS}{S} = r \, dt + \sigma_S \, dw_S \tag{9}$$

$$\frac{dK}{K} = - \lambda_K \, dt + \sigma_K \, dw_K \tag{10}$$

$$\frac{dD'}{D'} = (r - \lambda_K + \sigma_{SK})dt + \sigma_S \, dw_S + \sigma_K \, dw_K \tag{11}$$

We have used the fact that for a traded security like S, the risk adjusted drift (μ_S - λ_S) is equal to the riskfree interest rate r.

Using risk neutral valuation, we have

$$D(t) = \exp(-r(T-t)) \, E_t' \, (D'(T)) \tag{12}$$

Using Ito's lemma to compute the expectation of D'(T), we get:

$$\begin{aligned} D(t) &= \exp(-r(T-t)) \exp [(r - \lambda_K + \sigma_{SK})(T-t)] \, D'(t) \\ &= \exp [(- \lambda_K + \sigma_{SK})(T-t)] \, D'(t) \\ &= \exp [(- \lambda_K + \sigma_{SK})(T-t)] \, S(t)K(t) \end{aligned} \tag{13}$$

A naive valuation model would have suggested that D(t) should be equal to S(t)K(t), i.e., the current stock price times the current estimate of the conversion ratio. Our valuation model asserts that this naive estimate must be multiplied by an adjustment factor exp $[(-\lambda_K + \sigma_{SK})(T-t)]$.

The adjustment factor involves a covariance term σ_{SK} and a risk adjustment term λ_K. The covariance term arises because the expectation of S(T)K(T) is not equal to the product of the expectations of S(T) and K(T) when S and K are correlated. In a risk neutral world, if the interest rate is zero, D(t) would equal the simple expectation E[S(T)K(T)], and S(t) and K(t) would equal E[S(T)] and E[(K(T)] respectively. Even in this idealized world, D(t) is not equal to S(t)K(t) if S(T) and K(T) are correlated. The covariance adjustment is, therefore, of a purely mathematical nature and does not involve any financial concepts like time value of money or risk aversion.

The term involving λ_K is, on the other hand, purely a risk adjustment. While valuing the debenture, the risk averse investor would not use the expected conversion ratio but use some kind of certainty equivalent thereof. To make this clear, imagine a security whose payoff at time T is equal to the conversion ratio K(T). By Eqn 0, the expected return on this security is equal to $r + \lambda_K$, so that the price of this security at time t would be exp $[-(r+\lambda_K)(T-t)]$ K(t). In a risk neutral world the price of this imaginary security would be simply the discounted value of the expected payoff, i.e., exp $[(-r)(T-t)]$ K(t). The risk adjustment is the same as that observed in the case of the convertible debenture.

In general, we would expect both the covariance adjustment and the risk adjustment to be in the downward direction so that the market price D(t) would be significantly below the naive valuation S(t)K(t). The analyst who estimates the expected conversion ratio K(t) using all available information and multiplies this estimate by the current stock price S(t) to value the convertible would significantly overvalue the debenture as compared to the market. The analyst may, in fact, conclude wrongly that the market is perversely undervaluing the security.

Applying Ito's lemma, we can derive the dynamics of D(t):

$$\frac{dD}{D} = (\mu_S + \lambda_K)\, dt + \sigma_S\, dw_S + \sigma_K\, dw_K \qquad (14)$$

We see that the expected return on the security is equal to $\mu_S + \lambda_K$, while the expected return on the stock is only μ_S. The term λ_K represents compensation for conversion ratio risk.

It remains to see how the quantities σ_{SK} and λ_K can be estimated from the data which is available to a researcher. The difficulty, of course, is that the market's expectation of the conversion ratio K(t) is an unobservable variable as far as a researcher is concerned. Nevertheless, it is still possible to estimate σ_{SK} as follows. Consider the relationship between the stock returns and the debenture returns. The random component of the stock return is $\sigma_S\, dw_S$, while that of the debenture return is $\sigma_S\, dw_S + \sigma_K\, dw_K$. If we were to regress the debenture return on the stock return the regression slope would therefore be given by $(\sigma + \sigma_{SK})/\sigma = 1 + \sigma_{SK}/\sigma$. We can use this as well as an estimate of the stock volatility σ_S to estimate σ_{SK}. (The regression slope is of independent interest as it tells us the percentage change in the debenture price for a one percent change in the stock price. The above analysis shows that this slope is likely to be well below unity as σ_{SK} is negative. If the conversion ratio were pre-specified, this slope would equal unity).

However, the quantity λ_K cannot be estimated without some estimate of the expected conversion ratio K(t). If K(t) is known, then we can use D(t), S(t), K(t) and σ_{SK} to estimate λ_K by applying Eqn 0.

Refined Valuation Model

As stated earlier, the refined valuation model takes the value of the firm as a fundamental (state) variable and regards the price of the share as derived from it. The value of the firm (V) is a traded security as it consists of all the shares and debentures put together. We also now link our model with the well known Capital Asset Pricing Model (CAPM) by letting the dynamics of V depend on a market wide factor w_M and a residual factor w_R:

$$\frac{dV}{V} = \mu_V\, dt + \sigma_{VM}\, dw_M + \sigma_{VR}\, dw_R \qquad (15)$$

In this model, dw_M represents the unanticipated component of the market return (i.e., the return on the market portfolio), and dw_R represents the unanticipated component of the firm specific return on V. It is assumed that w_M and w_R are uncorrelated.

Instead of working with the conversion ratio K, it is more convenient to work with the reciprocal of the diluted share capital defined as follows:

Let n_S be the number of shares outstanding and n_D, the number of debentures outstanding. The diluted share capital after conversion is $n_S + K(T)*n_D$, and we define:

$$A(T) = \frac{1}{n_S + K(T)*n_D} \quad \text{so that} \quad K(T) = \frac{1 - A(T)*n_S}{A(T)*n_D} \qquad (16)$$

We let A(t) be the market's expectation at time t of A(T) and assume as in the earlier model that as t tends to T the expectation A(t) converges to the true value A(T). We assume the dynamics

$$\frac{dA}{A} = \sigma_A\, dw_A \qquad (17)$$

The share is now a derivative security defined by the payoff A(T)V(T). Define S'(t) = A(t)V(t). By Ito's lemma the dynamics of S' is given by:

$$\frac{dS'}{S'} = (\mu_V + \sigma_{AM} + \sigma_{AR})dt + \sigma_{VM}\, dw_M + \sigma_{VR}\, dw_R + \sigma_A\, dw_A \qquad (18)$$

where $\sigma_{AM} = \sigma_A\ \sigma_{VM}\ \rho_{AM}$, $\sigma_{AR} = \sigma_A\ \sigma_{VR}\ \rho_{AR}$, ρ_{AM} and ρ_{AR} are the instantaneous correlations of w_A with w_M and of w_A with w_R respectively.

The payoff to the debenture (which is the second derivative security) is given by

pV(T) - qS(T)

where $p = 1/n_D$ and $q = n_S/n_D$.

The risk adjusted system dynamics are as follows:

$$\frac{dV}{V} = r\ dt + \sigma_{VM}\, dw_M + \sigma_{VR}\, dw_R \qquad (19)$$

$$\frac{dA}{A} = -\ \lambda_A\ dt + \sigma_A\, dw_A \qquad (20)$$

$$\frac{dS'}{S'} = (r - \lambda_A + \sigma_{AM} + \sigma_{AR})dt + \sigma_{VM}\, dw_M + \sigma_{VR}\, dw_R + \sigma_A\, dw_A \qquad (21)$$

Using risk neutral valuation, we have

$$S(t) = \exp(-r(T-t))\ E_t'\ (S'(T)) \qquad (22)$$

Using Ito's lemma to compute the expectation of S'(T), we get:

$$\begin{aligned} S(t) &= \exp(-r(T-t))\ \exp\ [(r - \lambda_A + \sigma_{AM} + \sigma_{AR})(T-t)]\ S'(t) \\ &= \exp\ [(-\ \lambda_A + \sigma_{AM} + \sigma_{AR})(T-t)]\ S'(t) \qquad (23) \\ &= \exp\ [(-\ \lambda_A + \sigma_{AM} + \sigma_{AR})(T-t)]\ A(t)V(t) \end{aligned}$$

Once again, the valuation for S(t) differs from the naive valuation formula A(t)V(t) by a multiplicative factor $\exp\ [(-\ \lambda_A + \sigma_{AM} + \sigma_{AR})(T-t)]$.

Similarly,

$$\begin{aligned} D(t) &= \exp(-r(T-t))\ E'\ (pV(T) - qS(T)\ ) \\ &= p\ V(t) - q\ S(t) \qquad (24) \\ &= p\ V(t) - q\ A(t)V(t)\ \exp\ [(-\ \lambda_A + \sigma_{AM} + \sigma_{AR})(T-t)] \end{aligned}$$

As in the previous model, the market price differs from the naive valuation by a factor involving covariance adjustments and risk adjustments. As earlier, the covariance term $\sigma_{AM} + \sigma_{AR}$ is purely mathematical in nature. The risk adjustment is a more complex matter. It is now a double edged sword as uncertainty about conversion ratios affects shareholders and debenture holders in opposite ways.

Using Ito's lemma we can derive the dynamics of S(t) and D(t) as follows:

$$\frac{dS}{S} = (\mu_V + \lambda_A)dt + \sigma_{VM}\, dw_M + \sigma_{VR}\, dw_R + \sigma_A\, dw_A \tag{25}$$

$$\frac{dD}{D} = (\mu_V - \varphi\lambda_A)dt + \sigma_{VM}\, dw_M + \sigma_{VR}\, dw_R - \varphi\, \sigma_A\, dw_A \tag{26}$$

where $\varphi = qS/(pV-qS) = n_S S/n_D D$ is the ratio of the market value of all shares put together to the market value of all the debentures.

We see that the term λ_A enters the dynamics of D and S with opposite sign; this is quite natural given the diametrically opposite interests of shareholders who would like to see a low conversion ratio and debenture holders who would like to see a high conversion ratio. There is no a priori reason for expecting the risk adjustment λ_A to be positive or negative. The risk aversion of shareholders tends to make it positive, while the risk aversion of debenture holders tends to make it negative. From our understanding of the debenture and stock markets in India we would like to conjecture that λ_A would be negative tending to depress debenture prices.

In any case, we would expect the total adjustment of debenture prices including covariance and risk terms to be downward in direction. We, therefore, expect the market price of the debenture to be significantly lower than the naive valuation formula using the expected conversion ratios.

Once again, we shall show that the covariance terms σ_{AM} and σ_{AR} can be statistically estimated. If we regress the stock returns and debenture returns on the market return, the regression slopes (which are the CAPM betas) will be respectively $(\sigma_{VM} + \sigma_{AM})/\sigma$ and $(\sigma_{VM} - \varphi\sigma_{AM})/\sigma$. The difference in the betas is $(1+\varphi)\sigma_{AM}/\sigma$. This gives an estimate of σ_{AM} since σ is easily estimated. It is interesting to note that the beta of the debenture is expected to be significantly lower than that of the share, while in the case of a pre-specified conversion ratio, the betas should be identical. We now eliminate the impact of the market factor from the returns on V, S and D by taking residuals from regressions against the market return. If we look at the variances of these returns we obtain respectively:

σ_R
$\sigma_R + \sigma \quad + 2\sigma_R$
$\sigma_R + \varphi^2\sigma - 2\varphi\sigma_R$

This gives us a set of three linear equations in the three unknowns σ_R, σ and σ_R. Thus we get an estimate of σ_{AR}.

It is also interesting to examine the relationship between the residual (i.e. after eliminating market factor) returns on the stock and the debenture. It may be seen that the regression slope would be:

$$\frac{\sigma_R - \varphi\sigma + (1-\varphi)\sigma_R}{\sigma_R + \sigma + 2\sigma_R}$$

since σ_{VR}, φ, σ_A and σ_{AR} are all assumed positive, it follows that the regression slope will be below unity; in fact, this will be so even if σ_{AR} is zero. Under appropriate conditions, the slope could even turn negative. It is obvious that in the case of a fixed conversion ratio, the slope should be equal to unity.

As in the previous simpler model, it is not possible to estimate the risk adjustment parameter λ_A without having an estimate of A(t) or equivalently the conversion ratio K(t). Given such an estimate, however, we can use Eqn 0, the known values of S(t) and V(t) and the estimated vales of σ_{AM} and σ_{AR} to obtain an estimate of λ_A.