An Efficient Algorithm for Frequent Pattern Mining for Real-Time Business Intelligence Analytics in Dense Datasets

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Abstract

Finding frequent patterns from databases has been the most time consuming process in data mining tasks, like association rule mining. Frequent pattern mining in real-time is of increasing thrust in many business applications such as e-commerce, recommender systems, and supply-chain management and group decision support systems, to name a few. A plethora of efficient algorithms have been proposed till date, among which, vertical mining algorithms have been found to be very effective, usually outperforming the horizontal ones. However, with dense datasets, the performances of these algorithms significantly degrade. Moreover, these algorithms are not suited to respond to the real-time need. In this paper, we describe BDFS(b)-diff-sets, an algorithm to perform real-time frequent pattern mining using diff-sets and limited computing resources. Empirical evaluations show that our algorithm can make a fair estimation of the probable frequent patterns and reaches some of the longest frequent patterns much faster than the existing algorithms.

1. Introduction

In recent years, business intelligence systems are playing pivotal roles in fine-tuning business goals such as improving customer retention, market penetration, profitability and efficiency. In most cases, these insights are driven by analyses of historic data. Now the issue is, if the historic data can help us make better decisions, how real-time data can improve the decision making process [1].

Frequent pattern mining for large databases of business data, such as transaction records, is of great interest in data mining and knowledge discovery [2], since its inception in 1993, by Agrawal et al. In this paper, we assume that the reader knows the basic assumptions and terminologies of mining all frequent patterns.

Researchers have generally focused on the frequent pattern mining, as it is complex and the search space needed for finding all frequent itemsets is huge [2]. A number of efficient algorithms have been proposed in the last few years to make this search fast and accurate[3]. Among these, a number of effective vertical mining algorithms have been recently proposed, that usually outperforms horizontal approaches [4]. Despite many advantages of the vertical format, the methods tend to suffer, when the tid-list cardinality gets very large as in the case of dense datasets [4]. Again, these algorithms have limited themselves to either breadth first or depth first search techniques. Hence, most of the algorithms stop only after finding the exhaustive (optimal) set of frequent itemsets and do not promise to run under user defined real-time constraints and produce some satisficing (interesting sub-optimal) solutions due to their limiting characteristics[5, 6].

In this paper, we describe BDFS(b)-diff-sets (adopted from[5, 6]), a real-time frequent pattern mining algorithm which runs under limited execution time and has the capability of running under limited memory as well. BDFS(b)-diff-sets does not limit itself to either of breadth-first or a depth-first search, but uses a search technique, which is a good mix of the staged search and depth-first search (discussed later in section 4.1), adopted from [7]. We have adopted the diff-sets concept as introduced by [4] as it has been found to be very effective in cases of dense datasets.
In this paper, we also show the edge of BDFS(b)-diff-sets over existing efficient association mining algorithms such as Apriori [8], FP-Growth [9], Eclat [10] and dEclat [4], when it runs to completion and outputs exhaustive set of frequent patterns.

The rest of the paper is organized as follows. In the next section we present business issues of real-time frequent pattern mining in brief. In Section 3, we discuss a review of the previous work in association rule mining. In Section 4, we introduce algorithm BDFS(b)-diff-sets implemented using diff-sets[4]. Section 5 contains the empirical evaluation of our algorithm. Finally, we conclude the paper in Section 6.

2. Business Issues of Real-Time Frequent Pattern Mining

An offline analytic approach to data mining reflects sound practice because the data have to be cleaned, checked for accuracy, etc. However, in a scenario of cutthroat competition, the organizations cannot afford to show the attitude of not keeping abreast with the latest changing demands and trends of their customers and get satisfied with periodical data. They have to act on the latest data that is available to them to react not only to the fierce global competition, but also market products keeping in mind of the latest customer wishes. In such a scenario, the concept of a real-time enterprise has creped into the corporate boardrooms of a number of organizations.

Using up-to-date information, getting rid of delays, and using speed for competitive advantage is what the real-time enterprise is about [11].

Frequent pattern mining has been extensively used for market basket analysis of data, to find out the hidden patterns that lie in the transactional database. To promote a particular product, if a retailer decides to go for dynamic pricing or for dynamic discount, she must do it before the customer actually moves out of the store. Hence, the retailer cannot afford to make run on the huge dataset again and again to depict the correct association rule for a particular customer before she moves out of the store. Again, the strategy of making the association mining an offline task and refer to the patterns for a particular time period may also prove to be ineffective because the customer preference may considerably change over time. Hence, dynamic pricing or offering dynamic discounts will not be able to fetch the necessary returns from the customer(s), if the whole exercise is based on patterns that were obtained previously. With competition growing at a break-neck speed, organizations have started appreciating the real-time analysis and real-time decision making for the particular concerned customer [12]. The importances for real-time solutions have been felt more lately due to the introduction and development of online businesses (although for offline businesses as well, the thrust remains the same). Researchers [13] believe that real-time personalization technology will proactively offer a particular customer products and services that will fit into their need exactly. A real-time analytical engine will work in real-time, analyzing web clicks or sales rep interactions and matching them with the past purchasing history to make the offerings.

In cases of event based information management systems, as the example in the previous paragraph, current approaches of business intelligence systems using various data mining techniques make organizations face some serious latency problems, which they must overcome. These are: *data latency, analysis latency and decision latency*. The following exhibit will make the point clearer.

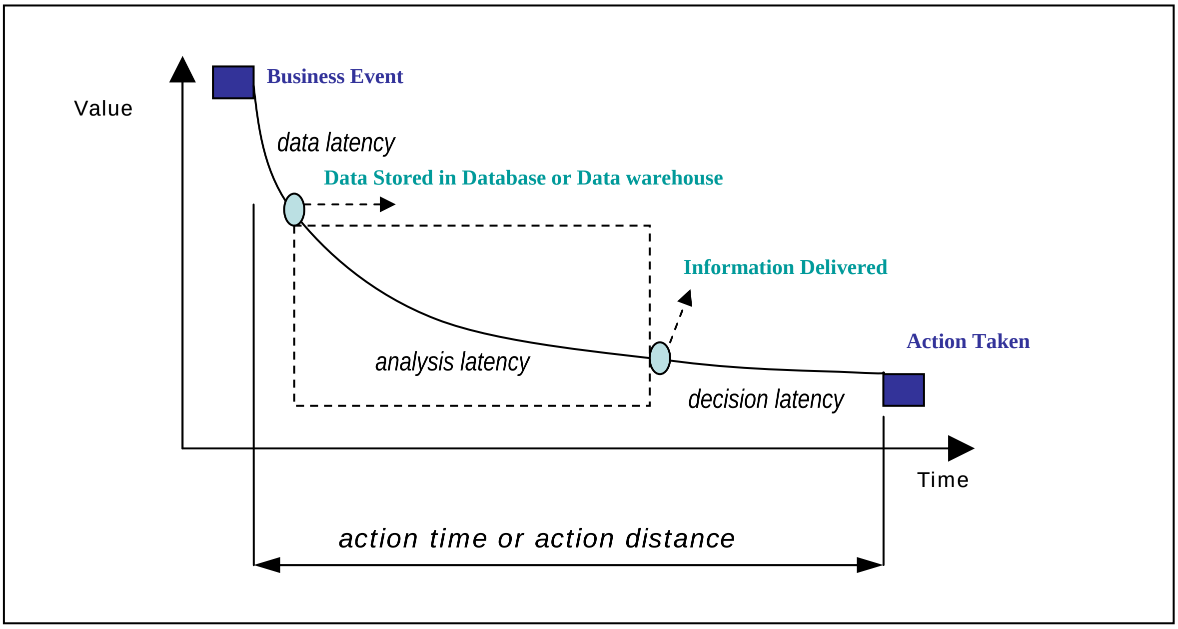

Exhibit 1. Framework for real-time business intelligence. Organizations must manage three distinct processes that create latency in an analytic environment to support real-time decision making. ***Source[14]***

Once a business event happens, users face *data latency*, meaning the time taken for various pre-processing steps for storing this data into the corresponding database or data warehouse. On this data, various analytic processes have to run for discovering the relevant information and delivering it to the right user for the purpose of decision making. This phase, referred to as analytic latency in Exhibit 1, refers to the time taken by various algorithms to run on the corresponding database or data warehouse. Once the information is delivered, the user may take some time before she can take any action on this delivered information. This is referred to as *decision latency*, in Exhibit 1. As pertinent from the above figure, the majority of the action time is caused due to the *analytic latency* only. Hence the major challenge to bye-pass these latencies and delivering *right information* to the *right user* within *right time* is the *analytic latency*. This means that the existing technologies hinder in responding to the real-time need of the business user due to their in-built limitations as they do not have the capability to respond to the real-time need. This real-time time bound, as described by various authors as *right time*, will vary from user-to-user and from industry to industry. In an research carried by TDWI (The Data Warehousing Institute), based on the responses of 383 respondents world wide, who have deployed various data mining related systems in organizations, it has been found that the major factors that create the bottle-neck of reducing the *analytic latency* and real-time business intelligence are lack of tools for doing real-time processing, immature technology and performance issues in Exhibit 2.

Lack of tools for doing real-time processing	35%
Immature technology	28%
Performance and scalability	24%

Exhibit 2. Obstacles to real-time business intelligence
Source[15]

There are numerous areas where real-time decision making plays a crucial role. These include areas like real-time customer relationship management [16-18], real-time supply chain management systems [19] real-time enterprise risk and vulnerability management [20], real-time stock management and vendor inventory [21], real-time recommender systems[22], real-time operational management with special applications in mission critical real-time information as is used in the airlines industry, real-time intrusion and real-time fraud detection [23], real-time negotiations and other areas like real-time dynamic pricing and discount offering to customers in real-time. More than that, real-time data mining will have tremendous importance in areas where a real-time decision can make the difference between life and death – mining patterns in medical systems.

3. Previous Work Done

A detailed discussion about the various algorithms of frequent pattern mining and their performance can be found in the literature surveys of frequent pattern mining [3, 24, 25]. Majority of the algorithms in this area have been classified according to their strategy to traverse the search space and by their strategy to determine the support values of the itemsets [24]. However, Su & Lin [26] have concluded that the most salient features of these algorithms are their *counting strategy*, *search direction* and *search strategy* (**Table 1**). Recently, a number of vertical mining algorithms have been proposed[4, 10, 27]. In a vertical database, each item is associated with its corresponding set of transactions where the particular item appears [4], called tid-list. However, in dense datasets, the method suffers since the intersection time becomes very high. Furthermore, the scalability of these algorithms gets affected, when the vertical tid-lists become too large for memory. Zaki [4] has introduced the concept of d*iff-sets,* that only keeps track of the differences in the tids of a candidate pattern from its generating frequent patterns. This diff-set implementation drastically cut down the size of the memory and tid-list intersections are done significantly faster (as diff-sets are a small fraction of the size of tid-lists).

Counting Strategy	**Search Direction**			
	Bottom-up		*Top-Down*	
	Search Strategy		**Search Strategy**	
	Depth-first	*Breadth-first*	*Depth-first*	*Breadth-first*
Counting	FP-Growth	Apriori		Top-Down
Intersection of tid-lists	Eclat	Partition		
Intersection of Diff-Sets	dEclat			

Table 1. Classification of prevaililng algorithms

4. BDFS(b)-diff-sets: An Efficient Technique of Frequent Pattern Mining In Real-Time Using Diff-Sets

4.1 Algorithm Basics

In this study, we propose a brute force algorithm BDFS(b)-diff-sets, which is a variant of the Block Depth First Search [7] and inducted into the domain of frequent pattern mining [5, 6]. Block Depth First Search is a search algorithm, based on a novel combination of the staged search and the depth first search [28]. As a result, it has the merits of both best-first search and the depth-first-branch-and-bound (DFBB) search [29], ,and at the same time, avoids bad features of both. BDFS(b)-diff-sets explores the given search space in stages. The search is conducted in a depth first manner, which ensures that patterns of greater length will be preferred over those of comparatively shorter lengths. We assume that a lower triangular frequency matrix M for a given database is created in a support-independent pre-processing step and kept in the hard-disk, which stores the support independent frequencies of all 1-length and 2-length patterns. Once the user specifies a desired support value, all frequent patterns of length 1 and 2 (meaning F(1) and F(2), where F(n) means frequent pattern of length-n) are obtained from M. Then BDFS(b)-diff-sets starts its search for frequent patterns of higher lengths from this point forward by intersecting the diff-set tid-lists of corresponding items. The most salient features of BDFS(b)-diff-sets are:(a) It conducts search in stages and uses back-tracking strategy to run to completion and ensure optimal solution. (b) It takes a block of candidate patterns b from a global pool, conducts the search by checking the frequency of these patterns in the database. It generates the possible candidate patterns (explained later with an example) of the next higher length from the currently known frequent patterns. These candidate patterns are continued to be explored in a systematic manner until all frequent patterns are generated. In this paper, we keep the block b variable and the value to be defined by the user using her knowledge and experience depending on the available computer memory (later in the paper, we have shown how the performance of BDFS(b)-diff-sets is affected with changing block size b) for purposes of academic curiosity to find how it affects the performance of the algorithm. A possible state space diagram of BDFS(b)-diff-sets is shown in. Fig. 1

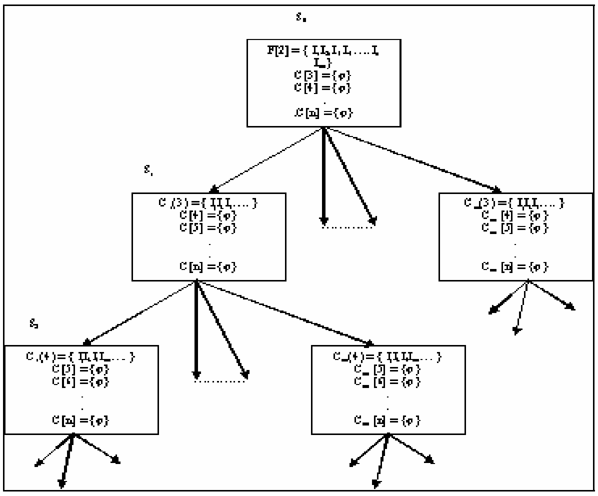

Fig. 1. State space representation of BDFS(b)-diff-sets

The initial state (or the root node) in the state-space is denoted by S_0, which contains the complete set of 2-length frequent patterns F(2). In $S_{0,}$, the set of all candidate patterns of length 3 or more are set to ϕ. In general, by the expansion of a node (which is a block of candidate patterns in this case) we mean:

i. Counting the support frequency of all candidate patterns in the state from the database by intersecting the diff-sets of the corresponding items.
ii. Generating the candidate patterns or patterns of border set of next higher level (explained later in the algorithm and its working through example).
iii. Arranging the candidate patterns according to their merits (explained later) and group them into blocks containing b-patterns each. If the block has empty space, it gets candidate patterns from the previous level. This can be handled using a global pool of candidate patterns that has been sorted in descending order of length. We resolve ties arbitrarily.

We have implemented this algorithm with diff-sets as proposed by [4] and have used the prefix based tree, called trie, data structure for implementing BDFS(b)-diff-sets.

4.2 Algorithm Details

Algorithm BDFS(b)-diff-sets:

Initialize the allowable execution time τ.
Let the initial search frontier contain all 3-length candidate patterns. Let this search frontier be stored as a global pool of candidate patterns. Initialize a set called Border Set to null.
Order the candidate patterns of the global pool according to their decreasing length (resolve ties arbitrarily). Take a group of most promising candidate patterns and put them in a block b of predefined size.

- *Expand (b)*

Expand (b: block of candidate patterns)
If not last_level
then
begin
$Expand_1(b)$
end.
$Expand_1(b)$:

1. *Count support for each candidate pattern in the block b by intersecting the diff-set list of the items in the database.*
2. *When a pattern becomes frequent, remove it from the block b and put it in the list of frequent patterns along with its support value. If the pattern is present in the Border Set increase its subitemset counter. If the subitemset counter of the pattern in Border Set is equal to its length move it to the global pool of candidate patterns.*
3. *Prune all patterns whose support values < given minimum support. Remove all supersets of these patterns from Border Set.*
4. *Generate all patterns of next higher length from the newly obtained frequent patterns at step 3. If all immediate subsets of the newly generated pattern are frequent then put the pattern in the global pool of candidate patterns else put it in the Border Set if the pattern length is > 3.*
5. *Take a block of most promising b candidate patterns from the global pool.*
6. *If block b is empty and no more candidate patterns left, output frequent patterns and exit.*
7. *Call Expand (b) if enough time is left in τ to expand a new block of patterns, else output frequent patterns and exit.*

Fig. 2. Algorithm BDFS(b)-diff-sets

5 Empirical Evaluation

Legend: T= Average size of transaction; I= Average size of the maximal potentially large itemset; D= No. of transactions in the database; N= Number of items.

To evaluate the performance of BDFS(b)-diffsets with on dense datasets, we have tested it on various dense datasets. This includes real-life dense datasets like CHESS, Connect-4, PUMSB and PUMSB*1and synthetic datasets like: T10I8D100K, T10I8D10K, T10I8D1K (N=1K). These datasets were generated using the IBM synthetic data generator2 [2]. The experiments were performed on a Red-Hat Linux machine with 1GB RAM and 20 GB HD with Pentium IV 2.24Ghz processor.

[1] These datasets are publicly available at http://fimi.cs.helsinki.fi/data/
[2] The data generator is available from http://www.almaden.ibm.com/cs/quest//syndata.html#assocSynData

5.1 Comparison of BDFS(b)-diff-sets with existing algorithms

In order to show how BDFS(b)-diff-sets performs on dense datasets, when it is run to generate all frequent patterns, we have chosen to compare it with dEclat3, Eclat4, FP-growth5 and Apriori6. Since FP-growth is known to be an order faster and scales better than Apriori[9], we have compared Apriori and BDFS(b)-diff-sets but for their number of patterns checked. In figures 3, 4, 5 and 6, we have compared the run-time of FP-Growth, dEclat and Eclat with BDFS(b)-diff-sets for Pumsb, T10I8D100K and Pumsb* respectively and found that BDFS(b)-diff-sets significantly out-performs all the three algorithms in these cases. In figure 7, we have tested the scalability of Eclat and dEclat and BDFS(b)-diff-sets. We have observed that all the algorithms are scalable with time and number of transactions in the database, but BDFS(b)-diff-sets takes strikingly much less time than dEclat, and Eclat over the same databases. Comparing the number of patterns being checked by Apriori and BDFS(b)-diff-sets, as shown in figure 8, it is found that BDFS(b)-diff-sets checks much lesser number of patterns than Apriori. The performance imperatives come from the efficient search strategy of the block depth first search that BDFS(b)-diff-sets utilizes and combines the power of the diff-sets approach. It is worth mentioning at this point that the codes we have obtained from the public domains are highly optimized in resprct to implementation.

5.2 Real-Time Performance of BDFS(b)-diff-sets

Figures 9, 10, 11 and 12 summarize the real-time behavior of BDFS(b)-diff-sets by depicting the percentage of frequent patterns generated with percentage execution time having F(1) & F(2) included and excluded in two respective curves. This we have done to show how the real-time performance is affected by the two-dimensional matrix M. It may be noted that the over all percentage of output is almost always ahead of percentage execution time. In figure 9, we find out that we have approximately 95% of the frequent patterns in 25% of completion time. We have also observed that our proposed algorithm perform quite well on real-life dense dataset connect-4. and highest length patterns can be obtained in lesser than 50% of total execution time.

Although it can be argued that all the existing frequent pattern mining algorithms will give some output if the execution is stopped at a user-defined time, but we have found that their performance

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[3] The dEclat code used for comparison is publicly available at http://www.cs.helsinki.fi/u/goethals/software/index.html

[4] The Eclat code used for comparison is publicly available at http://fuzzy.cs.uni-magdeburg.de/~borgelt/eclat.html

[5] The FP-growth code used for comparison is publicly available at www.cse.cuhk.edu.hk/~kdd/program.html

[6] The Apriori code used for comparison is publicly available at http://www.cs.helsinki.fi/u/goethals/software/index.html

in the real-time output is not promising as they use either a breadth-first or a depth-first search only and do not try to promise real-time performance. In figures 13, 14 and 15, we do a comparison of the real-time output of the existing algorithms. In all the cases, we find that BDFS(b)-diff-sets outperforms al existing techniques in providing real-time output. From figure 15, we find that BDFS(b)-diff-sets can provide 70% of the frequent patterns in just 40% of execution time. Whereas depth-first search techniques like FP-Growth and dEclat provides much lesser patterns corresponding to the given time. Its worth mentioning at this point that BDFS(b)-diff-sets takes much lesser time for complete execution as shown before. In this case, the percentage time taken for a particular algorithm is the slice of its own total execution time. Had the comparison been done in a scale of absolute time, the real-time performance edge pf BDFS(b)-diff-sets would have been much more prominent.

Figures 16-21 shows the performance of BDFS(b)-diff-sets when the block size is varied. We find that for smaller block size we get higher length patterns quickly. Figs 22 and 23 give a tabular representation of the actual output. From figure 22 we find that all F[15} patterns are found only in 34% of completion time.

6 Conclusion

In this paper, we have proposed an algorithm BDFS(b)-diff-sets, a brute force version of the Block Depth First Search(BDFS) [7] and implemented with diff-sets [4]. First we have compared the performance of BDFS(b)-diff-sets with dEclat, Eclat, FP-Growth and Apriori and shown that it compares well with others. Moreover, by adjusting its block size properly, BDFS(b)-diff-sets has the extra ability to run with limited available memory, which often becomes a point of concern in other algorithms. We have then shown that while running under real-time constraints it outputs large chunks of frequent patterns with fractional execution times.

We have made detailed performance evaluation based on empirical analysis using commonly used synthetic and real-life dense datasets. Thus, we have demonstrated that real-time frequent pattern mining can be done successfully using BDFS(b)-diff-sets. We believe this study will encourage use of AI heuristic search techniques in real-time frequent pattern mining

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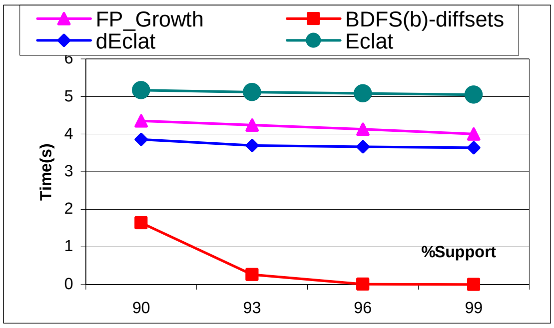

Fig. 3. Time comparison of FP-Growth, Eclat and dEclat with BDFS(b)-diffsets (b= 20880) on PUMSB, N=2113, T=74, D=49046

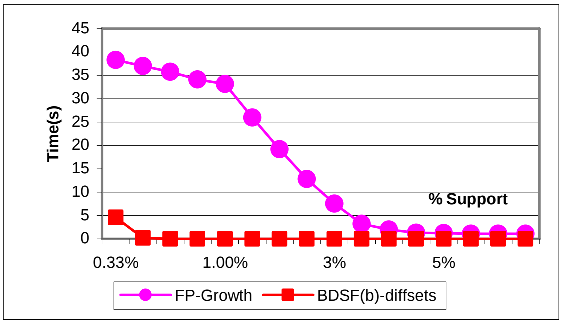

Fig. 4. Time comparison of FP-Growth with BDFS(b)-diffsets for T10I8D100K, b=100K. In most cases BDFS(b)-diffsets took in milli seconds only.

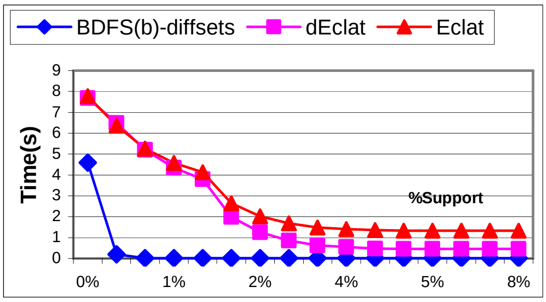

Fig. 5. Time comparison of Eclat and dEclat with BDFS(b)-diffsets for T10I8D100K, b=100K

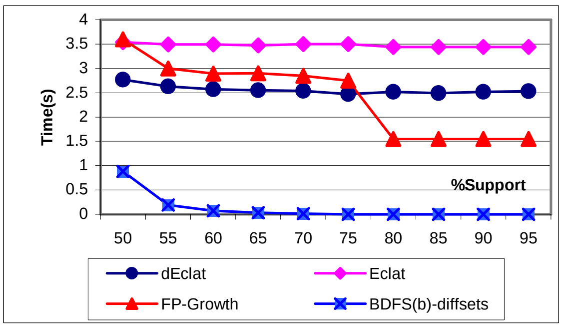

Fig. 6. Time comparison of FP-Growth, Eclat and dEclat with BDFS(b)-diffsets (b=2088K) for PUMSB*, N=2088 T= 50.5, D = 49046

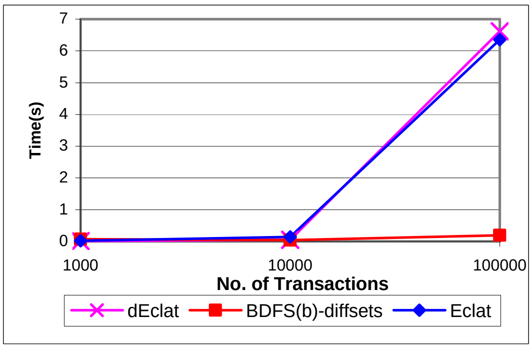

Fig. 7. Scalability evaluation of BDFS(b)-diffsets with Eclat and dEclat supp=0.5%, b = 100K for T10I8D1K,10K and 100K

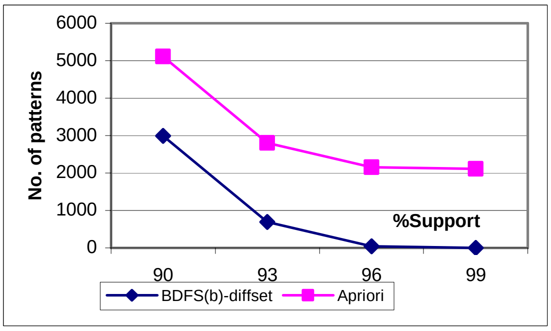

Fig. 8. Number of patterns checked by Apriori and BDFS(b)-diffsets (b=208800) for Pumsb, N=2113,T=74, D=49046, with varying support

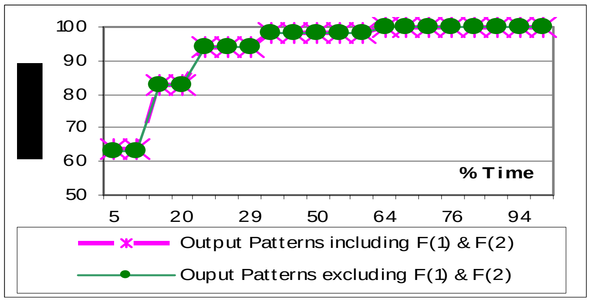

Fig. 9. Time-Patterns % of BDFS(b) for b=75K and 65% supp for Chess (N=75, T=37, D=3196)

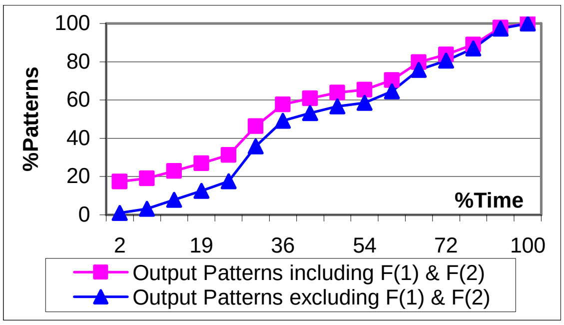

Fig. 10. Time-Patterns % for b=75K and 65% supp for T10I8D100K

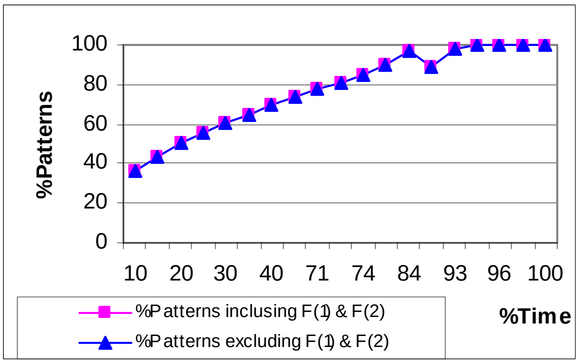

Fig. 11. Time-pattern% of BDFS(b), b=129, for 75% supp of Connect-4

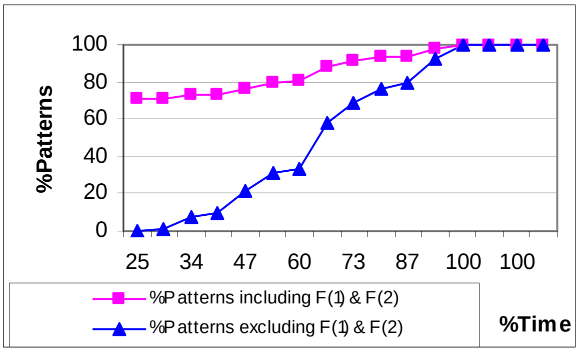

Fig.12. Time-pattern% of BDFS(b), b=1K, for T25I20D100K

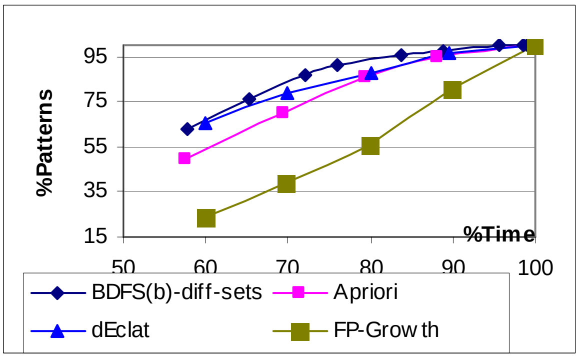

Fig. 13. Time-pattern% comparison of dEclat, Apriori, FP-Growth with BDFS(b)-diff-sets, b=1K, for 0.15% supp of T10I8D100K

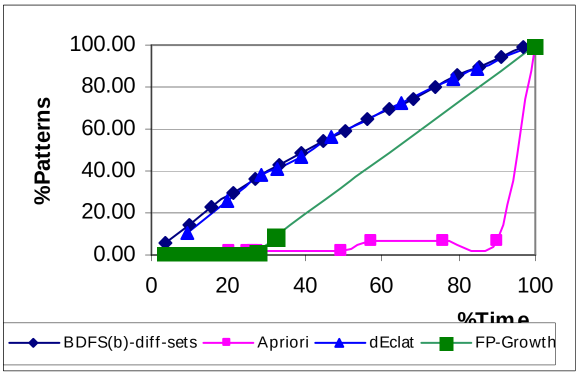

Fig. 14. Time-pattern% comparison of dEclat, Apriori, FP-Growth with BDFS(b)-diff-sets, b=2113, for 75% supp of PUMSB

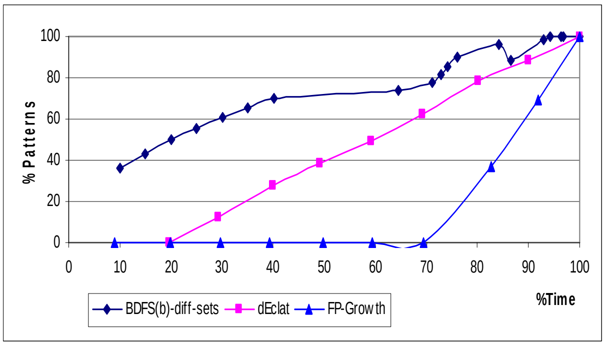

Fig.15. Time-pattern% comparison of dEclat, FP-Growth with BDFS(b)-diff-sets, b=380, for 75% supp of Connect-4

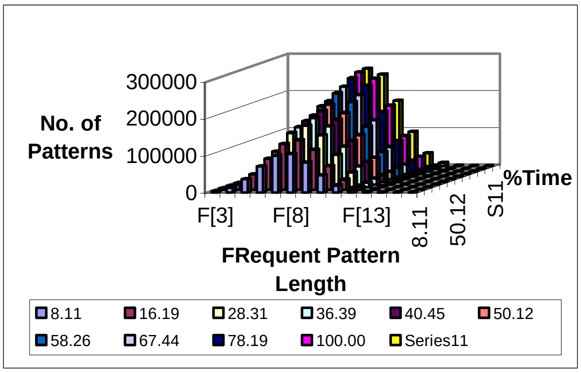

Fig. 16. Real-time output of frequent patterns by BDFS(b)-diff-sets, b=76, for 50% support of CHESS

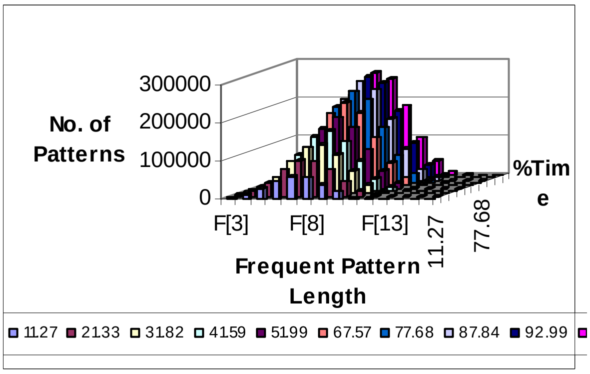

Fig. 17. Real-time output of frequent patterns by BDFS(b)-diff-sets, b=760, for 50% support of CHESS

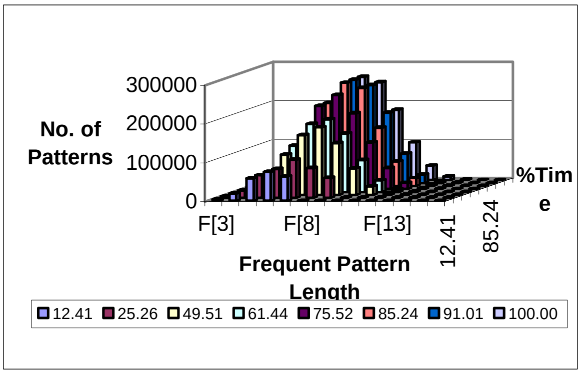

Fig. 18. Real-time output of frequent patterns by BDFS(b)-diff-sets, b=7600, for 50% support of CHESS

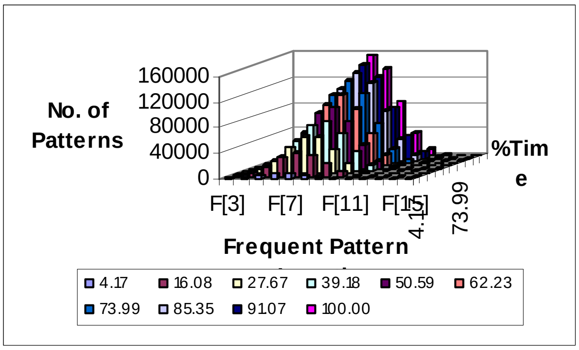

Fig. 19. Real-time output of frequent patterns by BDFS(b)-diff-sets, b=2113, for 75% support of PUMSB

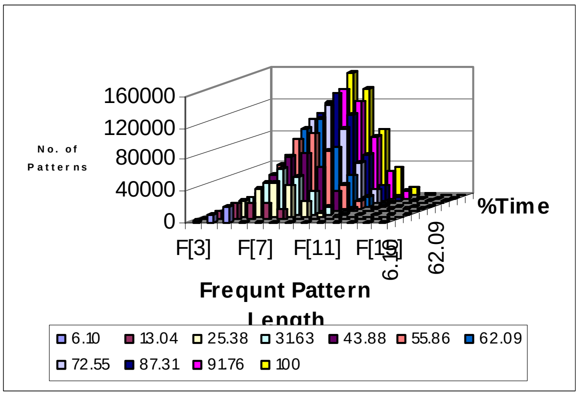

Fig. 20. Real-time output of frequent patterns by BDFS(b)-diff-sets, b=21130, for 75% support of PUMSB

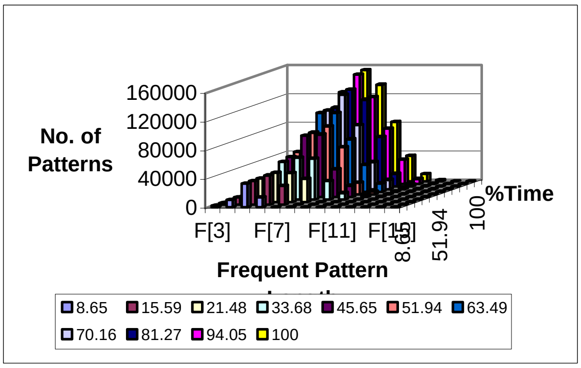

Fig. 21. Real-time output of frequent patterns by BDFS(b)-diff-sets, b=211300, for 75% support of PUMSB

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%Time	100	91	85	74	62	51	45	34	28	22	16	10	4
F[1]	27	27	27	27	27	27	27	27	27	27	27	27	27
F[2]	312	312	312	312	312	312	312	312	312	312	312	312	312
F[3]	2192	2192	2192	2068	2003	2003	2003	2003	2003	2003	2003	2003	2003
F[4]	10210	10210	10210	8992	8875	8629	8629	7320	7224	6847	5875	5098	2999
F[5]	32977	32370	30444	28262	27701	26009	25514	21224	20402	18717	15621	12238	5565
F[6]	76345	72996	67866	63903	61003	55906	52542	44037	40963	35870	28835	20554	7906
F[7]	128208	120747	111497	106074	95856	86421	79894	65110	58179	47548	37440	25085	7612
F[8]	155445	144715	135617	127761	110673	95111	85711	67868	57777	45049	32520	18881	5669
F[9]	135148	125864	121436	107890	89616	73018	63703	48178	37159	28139	19434	9716	2975
F[10]	83291	78102	77396	61661	49369	36766	32538	22193	15192	11559	7072	3220	1079
F[11]	35699	34219	34178	22678	17775	12344	11124	6297	3882	3018	1597	685	198
F[12]	10347	10141	10141	5240	3838	2692	2495	1237	596	493	234	96	29
F[13]	1951	1941	1941	718	511	367	348	179	55	48	24	7	2
F[14]	225	225	225	52	40	29	28	19	2	2	1	0	0
F[15]	13	13	13	1	1	1	1	1	0	0	0	0	0
C[3]	0	0	0	147	226	226	226	226	226	226	226	226	226
C[4]	0	0	0	285	118	366	366	1703	1800	2186	3202	4018	6233
C[5]	0	622	2664	35	8	303	812	100	440	280	254	509	1050
C[6]	0	0	599	661	929	99	1995	86	58	308	500	604	194
C[7]	0	0	626	215	0	333	104	727	428	0	0	87	0
C[8]	0	0	717	94	0	370	0	705	0	0	0	2275	0
C[9]	0	0	16	0	0	75	0	322	0	0	0	153	0
C[10]	0	0	0	0	0	1103	0	285	0	0	0	8	0
C[11]	0	0	0	0	0	65	0	282	0	0	0	0	0
C[12]	0	0	0	0	0	0	0	0	0	0	0	0	0
C[13]	0	0	0	0	0	0	0	0	0	0	0	0	0
C[14]	0	0	0	0	0	0	0	0	0	0	0	0	0
C[15]	0	0	0	0	0	0	0	0	0	0	0	0	0

Fig. 22.. Frequent output along with candidate sets of BDFS(b)-diff-sets for PUMSB data for 75% support and b=2113

%Time	100	97	93	84	76	65	40	35	30	25	20	15	10
F[1]	30	30	30	30	30	30	30	30	30	30	30	30	30
F[2]	379	379	379	379	379	379	379	379	379	379	379	379	379
F[3]	2757	2757	2686	2620	2598	2350	2350	2338	2301	2277	2073	2057	2036
F[4]	13220	13220	12697	12505	12401	10940	10906	10678	10477	10206	9123	8941	8697
F[5]	44734	44734	42863	42546	41767	36169	35915	34779	33787	32286	28707	27714	26392
F[6]	111195	111195	106965	106545	103230	88182	86946	83355	79942	75037	66833	63225	58543
F[7]	208182	208182	201756	200851	191694	162149	158082	149935	142010	131065	117571	108523	97045
F[8]	297893	297893	291124	288803	271305	227448	218317	204930	191741	174739	158197	141463	121414
F[9]	327845	327845	322868	318229	294473	244371	230101	213646	197589	178596	163222	140346	114785
F[10]	277176	277176	274660	268443	245184	200375	184318	169273	155131	139641	128500	105160	81295
F[11]	178425	178425	177597	171741	155304	123939	110874	100941	91801	82692	76212	58671	42283
F[12]	85860	85860	85702	81837	73595	56704	48964	44364	40122	36354	33287	23666	15575
F[13]	29910	29910	29897	28152	25330	18525	15276	13863	12488	11458	10329	6572	3833
F[14]	7136	7136	7136	6627	6014	4070	3147	2892	2610	2439	2152	1150	559
F[15]	1052	1052	1052	967	895	537	376	356	325	312	271	106	36
F[16]	76	76	76	70	67	32	19	19	18	18	16	3	0
F[17]	1	1	1	1	1	0	0	0	0	0	0	0	0
C[3]	0	0	76	153	176	447	447	459	502	527	738	754	775
C[4]	0	0	54	15	11	13	47	91	35	115	44	100	158
C[5]	0	0	33	18	18	51	0	16	2	82	1	61	4
C[6]	0	0	0	37	11	117	8	26	25	24	6	95	61
C[7]	0	0	0	52	17	12	35	89	22	0	120	6	0
C[8]	0	0	0	35	8	25	0	6	0	0	78	104	0
C[9]	0	0	0	0	0	2	0	54	0	0	0	9	0
C[10]	0	0	0	0	0	0	0	0	0	0	0	0	0

Fig. 23. Frequent output along with candidate sets of BDFS(b)-diff-sets for Connect-4 data for 75% support and b=129